

# 4

## Fuqizimi dhe rrënjëzimi

### 1. FUQIA ME EKSPONENTË NUMËR TË PLOTË

Le të njehsojmë prodhimin  $3 \cdot 3 \cdot 3 \cdot 3$ .

Vërejmë se numri 3 është përdorur katër herë si faktor.

Duke kryer shumëzimin përfundojmë se  $3 \cdot 3 \cdot 3 \cdot 3 = 81$ .

Një mënyrë më e përshtatshme për të shënuar prodhimin  $3 \cdot 3 \cdot 3 \cdot 3$  është  $3^4$ .

Pra

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3.$$

Shënimi  $3^4$  lexohet: **3 në fuqinë e 4-të**

Numri 3 quhet **baza** e fuqisë  $3^4$ .

Numri 4 quhet **eksponenti** i fuqisë  $3^4$ .

Në shprehjen  $3^4$  numri 4 tregon se sa herë baza 3 është përdorur si faktor.

Kështu

$3 \cdot 3 \cdot 3 \cdot 3$  - paraqet **formën e zgjeruar**

$3^4$  - paraqet **formën e fuqisë**

81 - paraqet **formën standarde**.

Duke u bazuar në shembullin e mësipërm, arrijmë tek rasti i përgjithshëm:

**Le të jetë  $n \in \mathbb{N}$ . Për çdo numër real  $a$  kemi:**

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n\text{-faktor}}$$

Në këtë rast numri  $a$  paraqet bazën kurse numri  $n$ -fuqinë.

**Shembulli 1.** Shprehjet vijuese të paraqiten në formë të fuqisë:

$$a) 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2;$$

$$b) \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2};$$

$$c) \frac{3}{4};$$

$$d) \underbrace{3 \cdot 3 \cdot \dots \cdot 3}_{n\text{-faktor}};$$

$$e) \underbrace{\frac{1}{3} \cdot \frac{1}{3} \cdot \dots \cdot \frac{1}{3}}_{m\text{-faktor}};$$

$$f) \underbrace{\left(-\frac{1}{4}\right) \cdot \left(-\frac{1}{4}\right) \cdot \dots \cdot \left(-\frac{1}{4}\right)}_{t\text{-faktor}}.$$

**Zgjidhja.**

a) Vërejmë se baza (numri) 2 është përdorur si faktor pesë herë. Pra kemi

$$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5.$$

b) Numri  $\frac{1}{2}$  është përdorur tri herë si faktor, pra kemi

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \left(\frac{1}{2}\right)^3.$$

c) Numri  $\frac{3}{4}$  është përdorur një herë si faktor, prandaj

$$\frac{3}{4} = \left(\frac{3}{4}\right)^1.$$

d) Numri 3 është përdorur  $n$  herë si faktor, prandaj

$$\underbrace{3 \cdot 3 \cdot \dots \cdot 3}_{n\text{-faktor}} = 3^n.$$

e) Numri  $\frac{1}{3}$  është përdorur  $m$  herë si faktor, prandaj

$$\underbrace{\frac{1}{3} \cdot \frac{1}{3} \cdot \dots \cdot \frac{1}{3}}_{m\text{-faktor}} = \left(\frac{1}{3}\right)^m.$$

f) Numri  $-\frac{1}{4}$  është përdorur  $t$  herë si faktor, prandaj

$$\underbrace{\left(-\frac{1}{4}\right) \cdot \left(-\frac{1}{4}\right) \cdot \dots \cdot \left(-\frac{1}{4}\right)}_{t\text{-faktor}} = \left(-\frac{1}{4}\right)^t.$$

**Shembulli 2.** Shprehjet vijuese të paraqiten në formën e zgjeruar.

$$\begin{array}{lll} a) 3^6; & b) (-2)^5; & c) \left(-\frac{1}{3}\right)^1; \\ d) 4^n; & e) 5^m; & f) (-6)^s. \end{array}$$

**Zgjidhja.**

a) Meqë baza është 3 kurse fuqia 6, atëherë numri 3 do të jetë 6 herë faktor, prandaj

$$3^6 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3.$$

b) Numri  $-2$  do të jetë 5 herë faktor, prandaj

$$(-2)^5 = (-2) \cdot (-2) \cdot (-2) \cdot (-2) \cdot (-2).$$

c) Numri  $\left(-\frac{4}{3}\right)$  do të jetë një herë faktor, prandaj

$$\left(-\frac{4}{3}\right)^1 = -\frac{4}{3}.$$

d) Numri 4 do të jetë  $n$  herë faktor, prandaj

$$4^n = \underbrace{4 \cdot 4 \cdot \dots \cdot 4}_{n\text{-faktor}}.$$

e) Numri 5 do të jetë  $m$ -herë faktor, pra

$$5^m = \underbrace{5 \cdot 5 \cdot \dots \cdot 5}_{m\text{-faktor}}.$$

f) Numri  $-6$  do të jetë  $s$ -herë faktor, prandaj

$$(-6)^s = \underbrace{(-6) \cdot (-6) \cdot \dots \cdot (-6)}_{s\text{-faktor}}.$$

**Detyra plotësuese**

Shprehjet vijuese të paraqiten në formë të fuqisë:

$$1. \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4}$$

$$2. 1\frac{1}{3} \cdot 1\frac{1}{3} \cdot 1\frac{1}{3}$$

$$3. 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7$$

$$4. (-0.4) \cdot (-0.4) \cdot (-0.4) \quad 5. \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right) \cdot \left(-\frac{2}{3}\right)$$

$$6. \underbrace{\frac{1}{2} \cdot \frac{1}{2} \cdot \dots \cdot \frac{1}{2}}_{n\text{-faktor}}$$

$$7. \underbrace{\left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right) \cdot \dots \cdot \left(-\frac{1}{3}\right)}_{m\text{-faktor}}$$

Shprehjet vijuese të paraqiten në formën e zgjeruar.

$$8. (-2)^5$$

$$9. \left(\frac{1}{3}\right)^4$$

$$10. (-0.5)^2$$

$$11. \left(-1\frac{1}{2}\right)^3$$

$$12. \left(-\frac{7}{3}\right)^1$$

$$13. \left(\frac{3}{2}\right)^n$$

$$14. 6^{n+1}$$

$$15. (-3)^{n+s}$$

Në shembujt e mëparshëm pamë se:

$$\left(\frac{3}{4}\right)^1 = \frac{3}{4}$$

Po ashtu

$$\left(-\frac{4}{3}\right)^1 = -\frac{4}{3}$$

Përfundojmë se:

**Për çdo numër real  $a$  vlen:**

$$a^1 = a.$$

**Shembulli 3.** Njehsoni:

$$a) 1^7;$$

$$b) (-1)^6;$$

$$c) (-1)^7.$$

**Zgjidhja.**

Kemi:

$$a) 1^7 = \underbrace{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}_{7\text{-faktor}} = 1.$$

$$b) (-1)^6 = \underbrace{(-1) \cdot (-1)}_1 \cdot \underbrace{(-1) \cdot (-1)}_1 \cdot \underbrace{(-1) \cdot (-1)}_1 = 1.$$

$$c) (-1)^7 = \underbrace{(-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot (-1)}_{6\text{-faktor}} \cdot (-1) = (-1)^6 \cdot (-1) = 1 \cdot (-1) = -1.$$

Përfundojmë se

$1^n = 1$ , për çfarëdo numri natyror  $n$ .

$(-1)^n = 1$ , nëse  $n$  është numër natyror çift.

$(-1)^n = -1$ , nëse  $n$  është numër natyror tek.

**Shembulli 4.** Le të njehsojmë:

$$a) (-3)^4, \quad b) (-3)^3.$$

**Zgjidhja.**

a) Numrin  $(-3)^4$  e paraqesim në formën e zgjeruar.

Merret:

$$\begin{aligned} (-3)^4 &= (-3) \cdot (-3) \cdot (-3) \cdot (-3) \\ &= (-1) \cdot 3 \cdot (-1) \cdot 3 \cdot (-1) \cdot 3 \cdot (-1) \cdot 3 \\ &= (-1) \cdot (-1) \cdot (-1) \cdot (-1) \cdot 3 \cdot 3 \cdot 3 \cdot 3 = (-1)^4 \cdot 3^4 \end{aligned}$$

Pak më sipër pamë se  $(-1)^4 = 1$ . Prandaj

$$(-3)^4 = 3^4.$$

b) Ngjashëm veprojmë me numrin  $(-3)^3$ .

Merret:

$$\begin{aligned} (-3)^3 &= (-3) \cdot (-3) \cdot (-3) = (-1) \cdot 3 \cdot (-1) \cdot 3 \cdot (-1) \cdot 3 \\ &= (-1) \cdot (-1) \cdot (-1) \cdot 3 \cdot 3 \cdot 3 = (-1)^3 \cdot 3^3 \end{aligned}$$

Meqë  $(-1)^3 = -1$  përfundojmë se:

$$(-3)^3 = -1 \cdot 3^3 = -3^3.$$

Japim këtë përgjithësim:

**Nëse  $n$  është numër natyror çift,  $(-a)^n = a^n$ .**

**Nëse  $n$  është numër natyror tek,  $(-a)^n = -a^n$ .**

**Shembulli 5.** Njehsoni:

$$\begin{aligned} a) & 1^{15}; & b) & 1^{12}; & c) & (-1)^{15}; & d) & (-1)^{14}; \\ e) & -(-1)^{13}; & f) & -(-1)^{12}; & g) & 1^{12} - (-1)^{11} + (-1)^{10}; \\ h) & 1^4 - (-1)^3 - (-1)^1 + (-1)^7. \end{aligned}$$

**Zgjidhja.**

a) Meqë 1 në çfarëdo fuqie është 1 atëherë  $1^{15} = 1$ .

b)  $1^{12} = 1$ .

c) Meqë 15 është numër tek, dhe meqë  $-1$  i ngritur në fuqinë numër tek është  $-1$  kemi  $(-1)^{15} = -1$ .

d)  $(-1)^{14} = 1$ .

e)  $-(-1)^{13} = -(-1) = 1$ .

f)  $-(-1)^{12} = -1$ .

g)  $1^{12} - (-1)^{11} + (-1)^{10} = 1 - (-1) + 1 = 1 + 1 + 1 = 3$ .

h)  $1^4 - (-1)^3 - (-1)^1 + (-1)^7 = 1 - (-1) - (-1) + (-1) = 1 + 1 + 1 - 1 = 2$ .

**Shembulli 6.** Njehsoni:

$$\begin{aligned} a) & (-2)^3; & b) & (-2)^4; & c) & -(-2)^5; \\ d) & \left(-\frac{1}{4}\right)^3; & e) & \left(-\frac{3}{2}\right)^2; & f) & \left(-\frac{1}{3}\right)^4; \\ g) & (-2)^3 - (-2)^4 + (-3)^4 + (-3)^3; \\ h) & \left(1\frac{1}{2}\right)^3 - \left(1\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^1; \\ i) & \left(-\frac{1}{4}\right)^2 - \left(-\frac{1}{4}\right) + \left(\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^3. \end{aligned}$$

**Zgjidhja.**

a) Meqë 3 është numër tek kemi  $(-2)^3 = -2^3 = -8$ .

b) Meqë 4 është numër çift kemi  $(-2)^4 = 2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$ .

c)  $-(-2)^5 = -(-2^5) = 2^5 = 32$ .

$$d) \left(-\frac{1}{4}\right)^3 = -\left(\frac{1}{4}\right)^3 = -\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = -\frac{1}{64}.$$

$$e) \left(-\frac{3}{2}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{3}{2} \cdot \frac{3}{2} = \frac{9}{4}.$$

$$f) \left(-\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^4 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{81}.$$

$$g) (-2)^3 - (-2)^4 + (-3)^4 + (-3)^3 = -2^3 - 2^4 + 3^4 + (-3^3) = -8 - 16 + 81 - 27 = 30.$$

$$h) \left(1\frac{1}{2}\right)^3 - \left(1\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^1 = \left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^2 - \frac{1}{2} = \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} - \frac{3}{2} \cdot \frac{3}{2} - \frac{1}{2}$$

$$= \frac{27}{8} - \frac{9}{4} - \frac{1}{2} = \frac{27 - 9 \cdot 2 - 1 \cdot 4}{8} = \frac{27 - 18 - 4}{8} = \frac{5}{8}.$$

$$i) \left(-\frac{1}{4}\right)^2 - \left(-\frac{1}{4}\right) + \left(\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^3 = \left(\frac{1}{4}\right)^2 + \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} - \left(-\left(\frac{1}{2}\right)^3\right)$$

$$= \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{16} + \frac{1}{4} + \frac{2}{8} = \frac{1}{16} + \frac{1}{4} + \frac{1}{4} = \frac{1}{16} + \frac{2}{4}$$

$$= \frac{1}{16} + \frac{1}{2} = \frac{1 + 1 \cdot 8}{16} = \frac{9}{16}.$$

### *Detyra plotësuese*

Njehsoni:

**16.**  $1^{13}$ .

**17.**  $1^{10}$ .

**18.**  $(-1)^7$ .

**19.**  $(-1)^6$ .

**20.**  $-(-1)^5$ .

**21.**  $-(-1)^4$ .

**22.**  $1^4 - (-1)^3 + (-1)^2$ .

**23.**  $1^8 - (-1)^9 - (-1) + (-1)^3$ .

Njehsoni

**24.**  $(-3)^2$ .

**25.**  $(-2)^5$ .

**26.**  $-(-3)^3$ .

**27.**  $(-0.5)^2$ .

**28.**  $\left(-\frac{1}{4}\right)^3$ .

**29.**  $-\left(-\frac{1}{2}\right)^3$ .

**30.**  $(-2)^2 - (-2)^3 + (-3)^3 - 3^2$ . **31.**  $\left(-1\frac{1}{2}\right)^3 - \left(-\frac{3}{2}\right)^2 + \left(\frac{1}{4}\right)^2$ .

**Shembulli 7.** Njehsoni  $2^3 \cdot 2^4$ .

**Zgjidhja.**

$$2^3 \cdot 2^4 = \underbrace{(2 \cdot 2 \cdot 2)}_{3\text{-faktor}} \cdot \underbrace{(2 \cdot 2 \cdot 2 \cdot 2)}_{4\text{-faktor}} = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{7\text{-faktor}}.$$

D.m.th.  $2^3 \cdot 2^4 = 2^{3+4} = 2^7$ .

Në përgjithësi vlen:

**Për çdo numër realë  $a$  vlen:**

$$a^m \cdot a^n = a^{m+n}, m, n \in \mathbb{N}.$$

**Shembulli 8.** Të njehsohet prodhimi i fuqive:

$$\begin{array}{lll} a) 3^3 \cdot 3^5; & b) 4^2 \cdot 4^3; & c) 5^2 \cdot 5^4; \\ d) x^2 \cdot x^9; & e) a \cdot a^9; & f) (a+b) \cdot (a+b)^4; \\ g) \left(\frac{x}{2}\right)^3 \cdot \left(\frac{x}{2}\right)^2; & h) \left(\frac{x+y}{3}\right)^0 \cdot \left(\frac{x+y}{3}\right)^5. \end{array}$$

**Zgjidhja.**

Zbatojmë vetinë e mësipërme:

$$a) 3^3 \cdot 3^5 = 3^{3+5} = 3^8.$$

$$b) 4^2 \cdot 4^3 = 4^{2+3} = 4^5.$$

$$c) 5^2 \cdot 5^4 = 5^{2+4} = 5^6.$$

$$d) x^2 \cdot x^9 = x^{2+9} = x^{11}.$$

$$e) \text{Meqë } a^1 = a, \text{ prandaj } a \cdot a^9 = a^1 \cdot a^9 = a^{1+9} = a^{10}.$$

$$f) (a+b) \cdot (a+b)^4 = (a+b)^1 \cdot (a+b)^4 = (a+b)^{1+4} = (a+b)^5.$$

$$g) \left(\frac{x}{2}\right)^3 \cdot \left(\frac{x}{2}\right)^2 = \left(\frac{x}{2}\right)^{3+2} = \left(\frac{x}{2}\right)^5.$$

$$h) \left(\frac{x+y}{3}\right)^0 \cdot \left(\frac{x+y}{3}\right)^5 = \left(\frac{x+y}{3}\right)^{0+5} = \left(\frac{x+y}{3}\right)^5.$$



**Detyra plotësuese**

Njehsoni:

**32.**  $3^2 \cdot 3^1$ .

**33.**  $2 \cdot 2^5$ .

**34.**  $2^0 \cdot 2^1$ .

**35.**  $A^3 \cdot A^5$ .

**36.**  $(a-1)^2 \cdot (a-1)^3$ .

**37.**  $(a-b)^2 \cdot (a-b)$ .

**38.**  $\left(\frac{x}{3}\right)^5 \cdot \left(\frac{x}{3}\right)^2$ .

**39.**  $\left(\frac{x-y}{2}\right)^0 \cdot \left(\frac{x-y}{2}\right)^5$ .

**40.**  $(x-1)^0 \cdot (x-1)^0$ .

**Mbani në mend**

Prodhimi  $x^2 \cdot x^5$  është baraz me  $x^7$ . Por  $x^7$  mund të paraqitet edhe në forma të tjera, p.sh.

$$x^7 = x \cdot x^2 \cdot x^4, x^7 = x^3 \cdot x^4, x^7 = x \cdot x^6.$$

Në disa raste, mënyra e shënimit lehtëson njehsimet me fuqi.

P.sh. nëse duam të njehsojmë  $2^9$  këtë mund ta bëjmë si vijon:

$$2^9 = 2^{3+3+3} = 2^3 \cdot 2^3 \cdot 2^3 = 8 \cdot 8 \cdot 8 = 64 \cdot 8 = 512.$$

Provoni të njehsoni  $2^9$  duke njehsuar prodhimin  $2^2 \cdot 2^7$ .

**Shembulli 9.** Njehsoni:

a)  $2^8$ ;

b)  $2^{10}$ ;

c)  $3^5$ ;

d)  $3^7$ ;

e)  $2^4 \cdot 3^2$ ;

f)  $4^3 \cdot 3^2$ .

**Zgjidhja.**

$$a) 2^8 = 2^{2+2+2+2} = 2^2 \cdot 2^2 \cdot 2^2 \cdot 2^2 = \underbrace{4 \cdot 4}_{16} \cdot \underbrace{4 \cdot 4}_{16} = 16 \cdot 16 = 256.$$

$$b) 2^{10} = 2^8 \cdot 2^2 = 256 \cdot 4 = 1024.$$

$$c) 3^5 = 3^{2+2+1} = 3^2 \cdot 3^2 \cdot 3 = 9 \cdot 9 \cdot 3 = 81 \cdot 3 = 243.$$

$$d) 3^7 = 3^5 \cdot 3^2 = 243 \cdot 9 = 2187.$$

$$e) 2^4 \cdot 3^2 = 2^2 \cdot 2^2 \cdot 3^2 = 4 \cdot 4 \cdot 9 = 16 \cdot 9 = 144.$$

$$f) 4^3 \cdot 3^2 = 4^{2+1} \cdot 3^2 = 4^2 \cdot 4 \cdot 9 = 16 \cdot 36 = 576.$$

**Shembulli 10.** Shprehjet e dhëna të shprehen si fuqi me bazë 2.

a)  $64 \cdot 2^3$ ;

b)  $16 \cdot 128$ ;

c)  $100 + 28$ .

**Zgjidhja.**

$$a) 64 \cdot 2^3 = 16 \cdot 4 \cdot 2^3 = 2^4 \cdot 2^2 \cdot 2^3 = 2^{4+2+3} = 2^9.$$

$$b) 16 \cdot 128 = 16 \cdot 2 \cdot 64 = 2^4 \cdot 2 \cdot 2^6 = 2^{4+1+6} = 2^{11}.$$

c) Nëse këtë rast mundohem ta zgjidhim sikur rastet paraprake do të vërejmë se një gjë e tillë është e pamundur sepse numrat 100,28 nuk mund të shprehen si fuqi të numrit 2.

Prandaj, së pari mbledhim numrat e dhënë dhe rezultatin që merret e paraqesim në formë të fuqive të numrit 2.

$$100 + 28 = 128 = 2 \cdot 64 = 2 \cdot 4 \cdot 16 = 2 \cdot 2^2 \cdot 2^4 = 2^7.$$

**Shembulli 11.** Shprehjet e dhëna të shprehen si fuqi me bazë 3.

$$a) 27 \cdot 3^2;$$

$$b) 81 \cdot 3^5;$$

$$c) 60 + 21.$$

**Zgjidhja.**

Veprojmë si në shembullin paraprak.

$$a) 27 \cdot 3^2 = 3^3 \cdot 3^2 = 3^5.$$

$$b) 81 \cdot 3^5 = 9 \cdot 9 \cdot 3^5 = 3^2 \cdot 3^2 \cdot 3^5 = 3^{2+2+5} = 3^9.$$

$$c) 60 + 21 = 81 = 9 \cdot 9 = 3^2 \cdot 3^2 = 3^{2+2} = 3^4.$$

**Shembulli 12.** Shprehjet e dhëna të shprehen si prodhim i fuqive me bazë 2 dhe 3.

$$a) 36 \cdot 72;$$

$$b) 108 \cdot 36;$$

$$c) 64 \cdot 6 \cdot 12.$$

**Zgjidhja.**

a) Së pari, pasi ta shprehim numrin 36 në faktor të thjesht merret:

$$36 = 2^2 \cdot 3^2.$$

Ngjashëm veprojmë me numrin 72. Merret:

$$72 = 2^3 \cdot 3^2.$$

Prandaj,

$$36 \cdot 72 = 2^2 \cdot 3^2 \cdot 2^3 \cdot 3^2 = 2^2 \cdot 2^3 \cdot 3^2 \cdot 3^2 = 2^5 \cdot 3^4.$$

b) Meqë  $108 = 36 \cdot 3 = \underbrace{2^2 \cdot 3^2}_{36} \cdot 3 = 2^2 \cdot 3^3$ ;  $36 = 2^2 \cdot 3^2$ , atëherë

$$108 \cdot 36 = 2^2 \cdot 3^3 \cdot 2^2 \cdot 3^2 = 2^4 \cdot 3^5.$$

c) Meqë  $64 = 2^6$ ;  $6 = 2 \cdot 3$ ;  $12 = 2^2 \cdot 3$  merret:

$$64 \cdot 6 \cdot 12 = 2^6 \cdot 2 \cdot 3 \cdot 2^2 \cdot 3 = 2^6 \cdot 2 \cdot 2^2 \cdot 3 \cdot 3 = 2^9 \cdot 3^2.$$

**Detyra plotësuese**

Njehsoni:

**41.**  $2^7$ .

**42.**  $2^9$ .

**43.**  $4^3$ .

**44.**  $3^6$ .

**45.**  $3^8$ .

**46.**  $2^3 \cdot 3$ .

**47.**  $3^4 \cdot 2^4$ ;

**48.**  $2^0 \cdot 2^3 \cdot 3^2$ .

Shprehjet e dhëna të shprehen si fuqi me bazën 2.

**49.**  $16 \cdot 2^5$ .

**50.**  $8 \cdot 2^3$ .

**51.**  $128 \cdot 128$ .

**52.**  $64 \cdot 32$ .

**53.**  $64 + 32$ .

**54.**  $30 + 34$ .

Shprehjet e dhëna të shprehen si fuqi me bazën 3.

**55.**  $81 \cdot 3$ .

**56.**  $9 \cdot 3^4$ .

**57.**  $243 \cdot 3^3$ .

**58.**  $27 \cdot 81$ .

**59.**  $27 + 81$ .

**60.**  $50 + 31$ .

Shprehjet e dhëna të shprehen si prodhim të fuqive me bazë 2 dhe 3.

**61.**  $24 \cdot 9$ .

**62.**  $48 \cdot 27$ .

**63.**  $48 \cdot 54$ .

**64.**  $24 \cdot 27 \cdot 36$ .

**65.**  $144 \cdot 12$ .

**66.**  $18 \cdot 36$ .

**Shembulli 13.** Të njehsohet herësi  $4^5 : 4^3$ .**Zgjidhja.**

$$\text{Kemi } 4^5 : 4^3 = \frac{4^5}{4^3} = \frac{4 \cdot 4 \cdot \cancel{4} \cdot \cancel{4} \cdot \cancel{4}}{\cancel{4} \cdot \cancel{4} \cdot \cancel{4}} = 4^2.$$

D.m.th.

$$4^5 : 4^3 = \frac{4^5}{4^3} = 4^{5-3} = 4^2.$$

Në përgjithësi vlen:

**Le të jetë  $a$  numër real i ndryshëm nga 0 dhe  $m, n \in N$ . Vlen:**

$$\frac{a^m}{a^n} = a^{m-n}.$$

Në qoftë se në rregullën e mësipërme merret  $m = n$  atëherë do të kemi:

$$a^{m-m} = \frac{a^m}{a^m}.$$

Meqë

$$a^{m-m} = a^0$$

dhe

$$\frac{a^m}{a^m} = 1$$

atëherë përfundojmë se:

**Për çdo numër real  $a$  të ndryshëm nga 0 ven:**

$$a^0 = 1.$$

**Shembulli 14.** Të njehsohet herësi i fuqive:

$$a) \frac{2^7}{2^4}; \quad b) 3^6 : 3; \quad c) a^5 : a^2;$$

$$d) a^{11} : a; \quad e) (a-b)^4 : (a-b)^3; \quad f) \left(\frac{a^2}{3}\right)^3 : \left(\frac{a^2}{3}\right)^0.$$

**Zgjidhja.**

$$a) \frac{2^7}{2^4} = 2^{7-4} = 2^3.$$

$$b) 3^6 : 3 = 3^6 : 3^1 = 3^{6-1} = 3^5.$$

$$c) a^5 : a^2 = a^{5-2} = a^3.$$

$$d) a^{11} : a = a^{11} : a^1 = a^{11-1} = a^{10}.$$

$$e) (a-b)^4 : (a-b)^3 = (a-b)^{4-3} = (a-b)^1 = a-b.$$

$$f) \left(\frac{a^2}{3}\right)^3 : \left(\frac{a^2}{3}\right)^0 = \left(\frac{a^2}{3}\right)^3 : 1 = \left(\frac{a^2}{3}\right)^3.$$

**Detyra plotësuese**

Njehsoni:

$$67. \frac{2^6}{2^4}.$$

$$68. \frac{3^5}{3^2}.$$

$$69. a^{12} : a^5.$$

$$70. \frac{(ax)^9}{(ax)^8}.$$

$$71. \frac{(b-x)^4}{(b-x)^4}.$$

$$72. \left(\frac{a+b}{2}\right)^5 : \left(\frac{a+b}{2}\right)^1.$$

$$73. \left(\frac{a^2}{4}\right)^4 : \left(\frac{a^2}{4}\right)^0 \qquad 74. \left(\frac{x-y}{3}\right)^5 : \left(\frac{x-y}{3}\right).$$

**Shembulli 15.** Njehsoni:

$$a) 4^7 : 4^3; \qquad b) 6^5 : 6^4; \qquad c) 16^2 : 4^3.$$

**Zgjidhja.**

$$a) 4^7 : 4^3 = 4^{7-3} = 4^4 = 4^2 \cdot 4^2 = 16 \cdot 16 = 256.$$

$$b) 6^5 : 6^4 = 6^{5-4} = 6^1 = 6.$$

$$c) 16^2 : 4^3 = \frac{16^2}{4^3} = \frac{16 \cdot 16}{4^3} = \frac{4^2 \cdot 4^2}{4^2 \cdot 4} = \frac{4^2}{4} = 4^{2-1} = 4^1 = 4.$$

**Detyra plotësuese**

Njehsoni:

$$75. \frac{4^7}{4^2}.$$

$$76. 6^5 : 6^6.$$

$$67. 7^9 : 7^8.$$

$$78. \frac{16}{4^2}.$$

$$79. \frac{64^2}{4^6}.$$

$$80. \frac{81}{3^3}.$$

**Shembulli 16.** Njehsoni

$$a) (4^3)^2. \qquad b) (4^3)^4.$$

*Çfarë mund të konkludojmë?*

**Zgjidhja.**

$$a) (4^3)^2 = 4^3 \cdot 4^3 = 4^{3+3} = 4^{3 \cdot 2} = 4^6.$$

$$b) (4^3)^4 = \underbrace{4^3 \cdot 4^3 \cdot 4^3 \cdot 4^3}_{4\text{-faktor}} = 4^{\overbrace{3+3+3+3}^{4\text{-mbledhës}}} = 4^{3 \cdot 4} = 4^{12}.$$

**Për çdo numër real  $a$  vlen:**

$$(a^m)^n = \underbrace{a^m \cdot a^m \cdot \dots \cdot a^m}_{n\text{-faktor}} = a^{\overbrace{m+m+\dots+m}^{n\text{-mbledhës}}} = a^{m \cdot n}, \quad m, n \in \mathbb{N}.$$

**Shembulli 17.** Fuqizoni fuqitë:

$$a) (3^2)^5; \qquad b) (a^2)^4; \qquad c) (a^5)^0; \qquad d) (x^5)^1;$$

$$e) (A^1)^{16}; \quad f) ((x-y)^3)^2; \quad g) \left( \left( \frac{x}{3} \right)^3 \right)^3.$$

**Zgjidhja.**

$$a) (3^2)^5 = 3^{2 \cdot 5} = 3^{10}.$$

$$b) (a^2)^4 = a^{2 \cdot 4} = a^8.$$

$$c) (a^5)^0 = a^{5 \cdot 0} = a^0 = 1.$$

$$d) (x^5)^1 = x^{5 \cdot 1} = x^5.$$

$$e) (A^1)^{16} = A^{1 \cdot 16} = A^{16}.$$

$$f) ((x-y)^3)^2 = (x-y)^{3 \cdot 2} = (x-y)^6.$$

$$g) \left( \left( \frac{x}{3} \right)^3 \right)^3 = \left( \frac{x}{3} \right)^{3 \cdot 3} = \left( \frac{x}{3} \right)^9.$$

**Shembulli 18.** Të shkruhen si fuqi me bazën  $a$  shprehjet:

$$a) (a^5 \cdot a^4)^3;$$

$$b) a \cdot (a^2)^5;$$

$$c) \frac{(a^3)^2}{(a^2)^3};$$

$$d) \frac{a \cdot (a^2 \cdot a^3 \cdot a^4)^4}{a^{12}}.$$

**Zgjidhja.**

$$a) (a^5 \cdot a^4)^3 = (a^{5+4})^3 = (a^9)^3 = a^{9 \cdot 3} = a^{27}.$$

$$b) a \cdot (a^2)^5 = a \cdot a^{2 \cdot 5} = a \cdot a^{10} = a^{1+10} = a^{11}.$$

$$c) \frac{(a^3)^2}{(a^2)^3} = \frac{a^{3 \cdot 2}}{a^{2 \cdot 3}} = \frac{a^6}{a^6} = a^{6-6} = a^0 = 1.$$

$$d) \frac{a \cdot (a^2 \cdot a^3 \cdot a^4)^4}{a^{12}} = \frac{a \cdot (a^{2+3+4})^4}{a^{12}} = \frac{a(a^9)^4}{a^{12}} = \frac{a \cdot a^{36}}{a^{12}} = \frac{a^{37}}{a^{12}} = a^{37-12} = a^{25}.$$

**Detyra plotësuese**

Fuqizoni fuqitë:

$$81. (3^4)^2.$$

$$82. (2^5)^0.$$

$$83. (2^6)^1.$$

$$84. (5^0)^4.$$

$$85. ((a+b)^2)^2. \quad 86. \left( \left( \frac{x}{2} \right)^3 \right)^4. \quad 87. ((x-1)^1)^2. \quad 88. ((a-1)^0)^1.$$

Të shkruhen si fuqi me me bazën  $a$  shprehjet:

$$89. (a^1 \cdot a^5)^4. \quad 90. (a^2 \cdot a^7)^5. \quad 91. (a^0 \cdot a^1)^0.$$

$$92. \frac{(a^5)^3}{(a^3)^5}. \quad 93. \frac{a \cdot (a^0 \cdot a^1)^4 \cdot a^5}{a^6 \cdot a^7}. \quad 94. \frac{a^0 \cdot a^9}{a^9 \cdot a^0}.$$

**Shembulli 19.** Të njehsohet  $(2 \cdot 3)^3$ . Çfarë mund të konkludojmë?

**Zgjidhja.**

$$(2 \cdot 3)^3 = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = (2 \cdot 2 \cdot 2)(3 \cdot 3 \cdot 3) = 2^3 \cdot 3^3.$$

Në përgjithësi vlen

$$(a \cdot b)^n = \underbrace{(a \cdot b) \cdot (a \cdot b) \cdot \dots \cdot (a \cdot b)}_{n\text{-faktor të } ab} = \underbrace{(a \cdot a \cdot \dots \cdot a)}_{n\text{-faktor të } a\text{-së}} \cdot \underbrace{(b \cdot b \cdot \dots \cdot b)}_{n\text{-faktor të } b\text{-së}} = a^n \cdot b^n.$$

Përfundojmë se:

**Për çdo dy numra realë  $a, b$  vlen:**

$$(a \cdot b)^n = a^n \cdot b^n, n \in N.$$

**Shembulli 20.** Shprehjet vijuese të paraqiten në formë të prodhimit të fuqive:

$$a) (2^3 \cdot 3^4)^5; \quad b) (2ab^2c^3)^4; \quad c) (-2x^3y^5z^7)^3.$$

**Zgjidhja.**

$$a) (2^3 \cdot 3^4)^5 = (2^3)^5 \cdot (3^4)^5 = 2^{3 \cdot 5} \cdot 3^{4 \cdot 5} = 2^{15} \cdot 3^{20}.$$

$$b) (2ab^2c^3)^4 = 2^4 \cdot a^4 \cdot (b^2)^4 \cdot (c^3)^4 = 2^4 \cdot a^4 \cdot b^{2 \cdot 4} \cdot c^{3 \cdot 4} = 2^4 \cdot a^4 \cdot b^8 \cdot c^{12}.$$

$$c) (-2x^3y^5z^7)^3 = (-2)^3 \cdot (x^3)^3 \cdot (y^5)^3 \cdot (z^7)^3 = (-2)^3 \cdot x^{3 \cdot 3} \cdot y^{5 \cdot 3} \cdot z^{7 \cdot 3}$$

$$= (-2)^3 \cdot x^9 \cdot y^{15} \cdot z^{21}.$$

**Detyra plotësuese**

Shprehjet vijuese të paraqiten në formë të prodhimit të fuqive:

$$95. (2^4 \cdot 3^2)^3. \quad 96. (2xy^2z^3)^5. \quad 97. (-3x^2 \cdot 5y^3)^4.$$

$$98. ((x+y)^2(x-y)^3)^4. \quad 99. (3^4 \cdot 4^3 \cdot 5^2 \cdot 7^1 \cdot 9^0)^5. \quad 100. (2 \cdot 3^2 \cdot 5^3 \cdot 7^4)^5.$$

**Shembulli 21.** Fuqia  $\left(\frac{3}{2}\right)^4$  të paraqitet si herës i fuqive. Çfarë mund të konkludojmë?

**Zgjidhja.**

$$\text{Kemi } \left(\frac{3}{2}\right)^4 = \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} \cdot \frac{3}{2} = \frac{3 \cdot 3 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 2} = \frac{3^4}{2^4}.$$

Në përgjithësi kemi:

$$\left(\frac{a}{b}\right)^n = \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \dots \cdot \frac{a}{b}}_{n\text{-faktor të } \frac{a}{b}} = \frac{\overbrace{a \cdot a \cdot \dots \cdot a}^{n\text{-faktor të } a\text{-së}}}{\underbrace{b \cdot b \cdot \dots \cdot b}_{n\text{-faktor të } b\text{-së}}} = \frac{a^n}{b^n}.$$

Përfundojmë se:

**Për çdo dy numra realë  $a, b$  ashtu që  $b \neq 0$ , vlen:**

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad n \in \mathbb{N}.$$

**Shembulli 22.** Të kryhen fuqizimet:

$$a) \left(\frac{2^3}{3^4}\right)^5; \quad b) \left(\frac{4x^2}{3y^3}\right)^5; \quad c) \left(\frac{(x-2y)^2}{(x+2y)^3}\right)^4.$$

**Zgjidhja.**

$$a) \left(\frac{2^3}{3^4}\right)^5 = \frac{(2^3)^5}{(3^4)^5} = \frac{2^{3 \cdot 5}}{3^{4 \cdot 5}} = \frac{2^{15}}{3^{20}}.$$

$$b) \left(\frac{4x^2}{3y^3}\right)^5 = \frac{(4x^2)^5}{(3y^3)^5} = \frac{4^5 \cdot (x^2)^5}{3^5 \cdot (y^3)^5} = \frac{4^5 \cdot x^{2 \cdot 5}}{3^5 \cdot y^{3 \cdot 5}} = \frac{4^5 \cdot x^{10}}{3^5 \cdot y^{15}}.$$

$$c) \left(\frac{(x-2y)^2}{(x+2y)^3}\right)^4 = \frac{((x-2y)^2)^4}{((x+2y)^3)^4} = \frac{(x-2y)^{2 \cdot 4}}{(x+2y)^{3 \cdot 4}} = \frac{(x-2y)^8}{(x+2y)^{12}}.$$

**Shembulli 23.** Njehsoni

$$a) \left(2\frac{1}{2}\right)^3 : \left(3\frac{1}{3}\right)^3; \quad b) \left(1\frac{1}{5} : \left(-\frac{36}{10}\right)\right)^3.$$



**Zgjidhja.**

$$a) \left(2\frac{1}{2}\right)^3 : \left(3\frac{1}{3}\right)^3 = \left(\frac{5}{2}\right)^3 : \left(\frac{10}{3}\right)^3 = \left(\frac{5}{2} : \frac{10}{3}\right)^3 = \left(\frac{5 \cdot 3}{2 \cdot 10}\right)^3 = \left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64};$$

$$b) \left(1\frac{1}{5} : \left(-\frac{36}{10}\right)\right)^3 = \left(\frac{6}{5} : \left(-\frac{36}{10}\right)\right)^3 = \left(\frac{\cancel{6}^1}{\cancel{5}_1} \cdot \left(-\frac{\cancel{10}^2}{\cancel{36}_6}\right)\right)^3 \\ = \left(-\frac{2}{6}\right)^3 = \left(-\frac{1}{3}\right)^3 = \frac{(-1)^3}{3^3} = -\frac{1}{27}.$$

**Detyra plotësuese**

Të kryhen veprimet me fuqi:

<b>101.</b> $(2^4)^7 : (3^2)^5$ .	<b>102.</b> $\left(\frac{2^5}{3^4}\right)^4$ .	<b>103.</b> $\left(\frac{4x^2y^4}{5z^5}\right)^3$ .
<b>104.</b> $\left(\frac{(x+a)^2}{(x-a)^3}\right)^5$ .	<b>105.</b> $\left(\frac{x^7}{y^8}\right)^0$ .	<b>106.</b> $\left(\frac{7^4 \cdot 8^5}{3^6}\right)^2$ .

<b>107.</b> $\left(1\frac{1}{3}\right)^4 : \left(1\frac{3}{4}\right)^5$ .	<b>108.</b> $\left(\left(1\frac{1}{5}\right)^2 \cdot \left(2\frac{1}{4}\right)^3\right)^4$ .
<b>109.</b> $\left(\left(1\frac{3}{4}\right)^2\right)^3 : \left(2\frac{1}{3}\right)^5$ .	<b>110.</b> $\left(2^3 : \left(2\frac{1}{2}\right)^4\right)^5$ .

**Shembulli 24.** Të njehsohet herësi  $4^3 : 4^5$ . Çfarë mund të konkludojmë?

**Zgjidhja.**

Në bazë të vetisë së mësuar më parë kemi:

$$4^3 : 4^5 = \frac{4^3}{4^5} = 4^{3-5} = 4^{-2}.$$

Por, ky rezultat paraqet risi në detyrat tona. Deri më tani patëm raste vetëm kur eksponenti është pozitiv.

Pra, sa është  $4^{-2}$ ?

Le të shqyrtojmë edhe një herë shprehjen  $4^3 : 4^5$ .

$$4^3 : 4^5 = \frac{4^3}{4^5} = \frac{\cancel{4} \cdot \cancel{4} \cdot \cancel{4}}{4 \cdot 4 \cdot \cancel{4} \cdot \cancel{4} \cdot \cancel{4}} = \frac{1}{4 \cdot 4} = \frac{1}{4^2}.$$

Pra

$$4^3 : 4^5 = 4^{-2}$$

dhe

$$4^3 : 4^5 = \frac{1}{4^2}.$$

Prandaj kemi  $4^{-2} = \frac{1}{4^2}$ .

Përfundojmë se:

**Për çdo numër real  $a$  të ndryshëm nga zero vlen:**

$$a^{-n} = \frac{1}{a^n}, n \in \mathbb{N}.$$

**Shembulli 25.** A është i saktë pohimi  $-2^{-3} = (-2)^{-3}$  ?

**Zgjidhja.**

Meqë  $-2^{-3} = -\frac{1}{2^3} = -\frac{1}{8}$  dhe  $(-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$  përfundojmë se pohimi

është i saktë.

**Shënim:** Proveni të kuptoni pse në përgjithësi nuk vlen pohimi

$$-a^{-n} = (-a)^{-n} ?$$

**Shembulli 26.** Të njehsohet:

$$a) (2^{1-3n})^{-3}; \quad b) \frac{x^{n-9}}{x^{2n-3}}.$$

**Zgjidhja.**

$$a) (2^{1-3n})^{-3} = 2^{(1-3n)(-3)} = 2^{-3+9n} = 2^{9n-3}.$$

$$b) \frac{x^{n-9}}{x^{2n-3}} = x^{n-9-(2n-3)} = x^{n-9-2n+3} = x^{-n-6}.$$

**Detyra plotësuese**

Të kryhen veprimet me fuqi:

111.  $\frac{4^5}{4^9}$ .

112.  $\frac{x^2}{x^7}$ .

113.  $\frac{(a+b)^3}{(a+b)^5}$ .

114.  $\left(\frac{4xy}{4^3 \cdot x^2 \cdot y^4}\right)^3$ .

115.  $\frac{(3^{-2}) \cdot (2^{-3})}{(2^{-4}) \cdot (3^{-3})}$ .

116.  $\left(\frac{4x^{-1}y^{-1}}{2^{-2}xy}\right)^{-1}$ .

117.  $(3^{1-n})^{-2}$ .

118.  $\frac{2^{n-3}}{2^{n-4}}$ .

119.  $\frac{4^{n-2}}{4^{3n}}$ .

110.  $\left(\frac{2^{n-4} \cdot 3^{n-3} \cdot 5^{n-2}}{2^{n-1} \cdot 3^{n+1} \cdot 5^{n-3}}\right)^{-2}$ .

Të thjeshtohen shprehjet:

**Shembulli 27.**  $\frac{10^3 \cdot 3^6 \cdot 6^5 \cdot (5^{-4})^4}{((-12)^{-4})^{-4} \cdot 5^{-12} \cdot 3^{-1} \cdot (-7)^0}$ .

**Zgjidhja.**

$$\begin{aligned} & \frac{10^3 \cdot 3^6 \cdot 6^5 \cdot (5^{-4})^4}{((-12)^{-4})^{-4} \cdot 5^{-12} \cdot 3^{-1} \cdot (-7)^0} = \frac{(2 \cdot 5)^3 \cdot 3^6 \cdot (2 \cdot 3)^5 \cdot 5^{-4 \cdot 4}}{(-12)^{-4 \cdot (-4)} \cdot 5^{-12} \cdot 3^{-1}} \\ & = \frac{2^3 \cdot 5^3 \cdot 3^6 \cdot 2^5 \cdot 3^5 \cdot 5^{-16}}{(-12)^{16} \cdot 5^{-12} \cdot 3^{-1}} = \frac{2^3 \cdot 2^5 \cdot 3^6 \cdot 3^5 \cdot 5^3 \cdot 5^{-16}}{(-1)^{16} \cdot 12^{16} \cdot 5^{-12} \cdot 3^{-1}} = \frac{2^{3+5} \cdot 3^{6+5} \cdot 5^{3-16}}{(3 \cdot 4)^{16} \cdot 5^{-12} \cdot 3^{-1}} \\ & = \frac{2^8 \cdot 3^{11} \cdot 5^{-13}}{3^{16} \cdot 4^{16} \cdot 5^{-12} \cdot 3^{-1}} = \frac{2^8 \cdot 3^{11} \cdot 5^{-13}}{(2^2)^{16} \cdot 3^{16} \cdot 3^{-1} \cdot 5^{-12}} = \frac{2^8 \cdot 3^{11} \cdot 5^{-13}}{2^{32} \cdot 3^{16-1} \cdot 5^{-12}} = \frac{2^8}{2^{32}} \cdot \frac{3^{11}}{3^{15}} \cdot \frac{5^{-13}}{5^{-12}} \\ & = 2^{8-32} \cdot 3^{11-15} \cdot 5^{-13-(-12)} = 2^{-24} \cdot 3^{-4} \cdot 5^{-1} = \frac{1}{2^{24}} \cdot \frac{1}{3^4} \cdot \frac{1}{5} = \frac{1}{2^{24} \cdot 3^4 \cdot 5}. \end{aligned}$$

**Shembulli 28.**  $\frac{2^3 \cdot (-7)^4 \cdot (3^{-2})^{-3}}{(4^{-2})^3 \cdot 49^2 \cdot (9^{-2})^{-1}}$ .

**Zgjidhja.**

$$\begin{aligned} & \frac{2^3 \cdot (-7)^4 \cdot (3^{-2})^{-3}}{(4^{-2})^3 \cdot 49^2 \cdot (9^{-2})^{-1}} = \frac{2^3 \cdot (-1)^4 \cdot 7^4 \cdot 3^{-2 \cdot (-3)}}{4^{-2 \cdot 3} \cdot (7^2)^2 \cdot 9^{-2 \cdot (-1)}} = \frac{2^3 \cdot 7^4 \cdot 3^6}{4^{-6} \cdot 7^4 \cdot 9^2} \\ & = \frac{2^3 \cdot 3^6}{(2^2)^{-6} \cdot (3^2)^2} = \frac{2^3 \cdot 3^6}{2^{-12} \cdot 3^4} = \frac{2^3}{2^{-12}} \cdot \frac{3^6}{3^4} = 2^{3-(-12)} \cdot 3^{6-4} = 2^{15} \cdot 3^2. \end{aligned}$$

**Shembulli 29.**  $\frac{81 \cdot x^{-4} \cdot y^{-3} \cdot (z^{-2})^{-1}}{27 \cdot x^{-7} \cdot (y^3)^{-2} \cdot (z^{-1})^2}$ .

**Zgjidhja.**

$$\frac{81 \cdot x^{-4} \cdot y^{-3} \cdot (z^{-2})^{-1}}{27 \cdot x^{-7} \cdot (y^3)^{-2} \cdot (z^{-1})^2} = \frac{3^4 \cdot x^{-4} \cdot y^{-3} \cdot z^{-2(-1)}}{3^3 \cdot x^{-7} \cdot y^{3(-2)} \cdot z^{-1 \cdot 2}} = \frac{3 \cdot x^{-4} \cdot y^{-3} \cdot z^2}{x^{-7} \cdot y^{-6} \cdot z^{-2}}$$

$$= 3 \cdot x^{-4-(-7)} \cdot y^{-3-(-6)} \cdot z^{2-(-2)} = 3 \cdot x^3 \cdot y^3 \cdot z^4.$$

**Shembulli 30.**  $\left(\frac{a^{-2}}{b^{-4}}\right) : \left(\frac{a^{-5}}{b^{-7}}\right)^{-3}.$

**Zgjidhja.**

$$\left(\frac{a^{-2}}{b^{-4}}\right) : \left(\frac{a^{-5}}{b^{-7}}\right)^{-3} = \left(\frac{a^{-2}}{b^{-4}}\right) : \frac{(a^{-5})^{-3}}{(b^{-7})^{-3}} = \frac{a^{-2}}{b^{-4}} : \frac{a^{-5(-3)}}{b^{-7(-3)}} = \frac{a^{-2}}{b^{-4}} : \frac{a^{15}}{b^{21}} = \frac{a^{-2}}{b^{21}}$$

$$= \frac{a^{-2} \cdot b^{21}}{a^{15} \cdot b^{-4}} = a^{-2-15} \cdot b^{21-(-4)} = a^{-17} \cdot b^{25}.$$

**Shembulli 31.** Të thjeshtohet shprehja  $A = \frac{x^3 \cdot y^{-2} \cdot (x^{-1} \cdot y^{-3})^4 \cdot x \cdot (y^{-3})^2}{(x^2 \cdot y^{-3})^2 \cdot (x \cdot y^{-3})^2}$

dhe të njehsohet vlera e saj nëse  $x = 4, y = \frac{1}{2}$ .

**Zgjidhja.**

$$A = \frac{x^3 \cdot y^{-2} \cdot (x^{-1} \cdot y^{-3})^4 \cdot x \cdot (y^{-3})^2}{(x^2 \cdot y^{-3})^2 \cdot (x \cdot y^{-3})^2} = \frac{x^3 \cdot y^{-2} \cdot (x^{-1})^4 \cdot (y^{-3})^4 \cdot x \cdot (y^{-3})^2}{(x^2)^2 \cdot (y^{-3})^2 \cdot x^2 \cdot (y^{-3})^2}$$

$$= \frac{x^3 \cdot y^{-2} \cdot x^{-4} \cdot y^{-12} \cdot x \cdot y^{-6}}{x^4 \cdot y^{-6} \cdot x^2 \cdot y^{-6}} = \frac{x^{3-4+1} \cdot y^{-2-12-6}}{x^{4+2} \cdot y^{-6-6}} = \frac{x^0 \cdot y^{-20}}{x^6 \cdot y^{-12}}$$

$$= x^{-6} \cdot y^{-20-(-12)} = x^{-6} \cdot y^{-8}.$$

Meqë  $x = 4, y = \frac{1}{2}$  merret

$$A = (4^{-6}) \cdot \left(\frac{1}{2}\right)^{-8} = \frac{1}{4^6} \cdot (2^{-1})^{-8} = \frac{1}{4^6} \cdot 2^8 = \frac{2^8}{(2^2)^6} = \frac{2^8}{2^{12}} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}.$$

**Shembulli 32.** Të thjeshtohet shprehja

$$\left(\frac{x + x^{-2}}{x^{-2} - x^{-1} + 1} + \frac{1 + x^{-1}}{x^{-2} + 2x^{-1} + 1}\right) \cdot \frac{1 + x^{-1}}{1 - x^{-1}}.$$

**Zgjidhja.**

$$\begin{aligned}
& \left( \frac{x+x^{-2}}{x^{-2}-x^{-1}+1} + \frac{1+x^{-1}}{x^{-2}+2x^{-1}+1} \right) \cdot \frac{1+x^{-1}}{1-x^{-1}} = \left( \frac{x+\frac{1}{x^2}}{\frac{1}{x^2}-\frac{1}{x}+1} + \frac{1+\frac{1}{x}}{\frac{1}{x^2}+2\cdot\frac{1}{x}+1} \right) \cdot \frac{1+\frac{1}{x}}{1-\frac{1}{x}} \\
& = \left( \frac{\frac{x^3+1}{x^2}}{\frac{1-x+x^2}{x^2}} + \frac{\frac{x+1}{x}}{\frac{1+2x+x^2}{x^2}} \right) \cdot \frac{\frac{x+1}{x}}{\frac{x-1}{x}} = \left( \frac{x^3+1}{x^2-x+1} + \frac{x(x+1)}{x^2+2x+1} \right) \cdot \frac{x+1}{x-1} \\
& = \left( \frac{(x+1)(x^2-x+1)}{x^2-x+1} + \frac{x(x+1)}{(x+1)^2} \right) \cdot \frac{x+1}{x-1} \\
& = \left( (x+1) + \frac{x}{x+1} \right) \cdot \frac{x+1}{x-1} = \frac{(x+1)^2+x}{x+1} \cdot \frac{x+1}{x-1} = \frac{x^2+2x+1+x}{x-1} = \frac{x^2+3x+1}{x-1}.
\end{aligned}$$

## 2. RRËNJËZIMI – FUQIA ME EKSPONENT NUMËR RACIONAL

Le të jetë  $a$  numër real dhe  $n$  numër natyror.

Zgjidhja e barazimit

$$x^n = a \quad (1)$$

sipas  $x$  (nëse ekziston) quhet rrënjë e  $n$  – të e numrit  $a$ .

Shënohet

$$x = (\sqrt[n]{a})^n = a \quad (2)$$

(këtu supozohet se  $\sqrt[n]{a}$  ekziston).

Nëse  $a$  është numër i çfarëdoshëm real, atëherë

$$\sqrt{a^2} = |a| \quad (3)$$

Nëse  $a \geq 0$  dhe  $m, n$  numra natyrorë, atëherë

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}. \quad (4)$$

Le të jetë  $a, b$  numra realë pozitiv dhe  $m, n$  numra natyrorë, atëherë

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad (5)$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad (6)$$

**Shembulli 1.** Njehsoni:

$$a) \sqrt{36} + 4\sqrt{49} + \sqrt[3]{8} - \sqrt[5]{32} + \sqrt[3]{1} - \sqrt[3]{-1};$$

$$b) \sqrt{\frac{16}{9}} - \sqrt[3]{\frac{1}{27}} + \sqrt[3]{64} - \sqrt{25} + \sqrt{(-3)^2};$$

$$c) \left( \sqrt{\frac{25}{36}} + \sqrt[3]{-8} \right) + (\sqrt{9} \cdot \sqrt[3]{-27} \cdot \sqrt[5]{-32}).$$

**Zgjidhja.**

$$\begin{aligned}
 a) \quad \sqrt{36} + 4\sqrt{49} + \sqrt[3]{8} - \sqrt[3]{32} + \sqrt[3]{1} - \sqrt[3]{-1} &= 6 + 4 \cdot 7 + \sqrt[3]{2^3} - \sqrt[3]{2^5} + 1 - (-1) \\
 &= 6 + 28 + 2 - 2 + 2 = 36.
 \end{aligned}$$

$$\begin{aligned}
 b) \quad \sqrt{\frac{16}{9}} - \sqrt[3]{\frac{1}{27}} + \sqrt[3]{64} - \sqrt{25} + \sqrt{(-3)^2} &= \frac{\sqrt{16}}{\sqrt{9}} - \frac{\sqrt[3]{1}}{\sqrt[3]{27}} + \sqrt[3]{4^3} - 5 + |-3| \\
 &= \frac{4}{3} - \frac{1}{3} + 4 - 5 + 3 = 3.
 \end{aligned}$$

$$\begin{aligned}
 c) \quad \left( \sqrt{\frac{25}{36}} + \sqrt[3]{-8} \right) + (\sqrt{9} \cdot \sqrt[3]{-27} \cdot \sqrt[5]{-32}) &= \left( \frac{\sqrt{25}}{\sqrt{36}} + \sqrt[3]{(-2)^3} \right) + (3 \cdot \sqrt[3]{(-3)^3} \cdot \sqrt[5]{(-2)^5}) \\
 &= \left( \frac{5}{6} + (-2) \right) + (3(-3)(-2)) = \frac{101}{6}.
 \end{aligned}$$

Njehsoni:

$$\text{Shembulli 3. } a) \sqrt{8} \cdot \sqrt{32}; \quad b) 3\sqrt{6} \cdot 2\sqrt{8}; \quad c) \sqrt{x^{m+2}} \cdot \sqrt{x^{m-2}}.$$

**Zgjidhja.**

$$\begin{aligned}
 a) \quad \sqrt{8} \cdot \sqrt{32} &= \sqrt{4 \cdot 2} \cdot \sqrt{16 \cdot 2} = \sqrt{4} \cdot \sqrt{2} \cdot \sqrt{16} \cdot \sqrt{2} = 2 \cdot \sqrt{2} \cdot 4 \cdot \sqrt{2} \\
 &= 8(\sqrt{2})^2 = 8 \cdot 2 = 16.
 \end{aligned}$$

$$\begin{aligned}
 b) \quad 3\sqrt{6} \cdot 2\sqrt{8} &= 3\sqrt{2 \cdot 3} \cdot 2\sqrt{4 \cdot 2} = 3\sqrt{2} \cdot \sqrt{3} \cdot 2\sqrt{4} \cdot \sqrt{2} = 6(\sqrt{2})^2 \cdot \sqrt{3} \cdot 2 \\
 &= 12 \cdot 2 \cdot \sqrt{3} = 24\sqrt{3}.
 \end{aligned}$$

$$c) \quad \sqrt{x^{m+2}} \cdot \sqrt{x^{m-2}} = \sqrt{x^{m+2} \cdot x^{m-2}} = \sqrt{x^{m+2+m-2}} = \sqrt{x^{2m}} = \sqrt{(x^m)^2} = |x^m|.$$

$$\text{Shembulli 4. } \left( 3^{\frac{1}{3}} + 2^{\frac{1}{3}} \right) \cdot \left( 3^{\frac{2}{3}} - 6^{\frac{1}{3}} + 2^{\frac{2}{3}} \right).$$

**Zgjidhja.**

$$\begin{aligned}
 \left( 3^{\frac{1}{3}} + 2^{\frac{1}{3}} \right) \cdot \left( 3^{\frac{2}{3}} - 6^{\frac{1}{3}} + 2^{\frac{2}{3}} \right) &= (\sqrt[3]{3} + \sqrt[3]{2})(\sqrt[3]{3^2} - \sqrt[3]{6} + \sqrt[3]{2^2}) \\
 &= (\sqrt[3]{3} + \sqrt[3]{2})(\sqrt[3]{3^2} - \sqrt[3]{3} \cdot \sqrt[3]{2} + \sqrt[3]{2^2}) = (\sqrt[3]{3})^3 + (\sqrt[3]{2})^3 = 5.
 \end{aligned}$$

Të racionalizohet emëruesi i thyesave:

**Shembulli 5.** a)  $\frac{1}{\sqrt{3}}$ ;      b)  $\frac{10}{\sqrt{5}}$ ;      c)  $\frac{1}{\sqrt[3]{3}}$ ;      d)  $\frac{2}{\sqrt[5]{7}}$ .

**Zgjidhja.**

$$a) \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3^2}} = \frac{\sqrt{3}}{3};$$

$$b) \frac{10}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{\sqrt{5^2}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5};$$

$$c) \frac{1}{\sqrt[3]{3}} = \frac{1}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}} = \frac{\sqrt[3]{9}}{\sqrt[3]{3^3}} = \frac{\sqrt[3]{9}}{3};$$

$$d) \frac{2}{\sqrt[5]{7}} = \frac{2}{\sqrt[5]{7}} \cdot \frac{\sqrt[5]{7^4}}{\sqrt[5]{7^4}} = \frac{2 \cdot \sqrt[5]{7^4}}{\sqrt[5]{7^5}} = \frac{2\sqrt[5]{7^4}}{7}.$$

**Shembulli 6.** a)  $\frac{1}{\sqrt{7}-2}$ ;      b)  $\frac{3-\sqrt{2}}{3+\sqrt{2}}$ ;      c)  $\frac{1}{2\sqrt{3}+1}$ .

**Zgjidhja.**

$$a) \frac{1}{\sqrt{7}-2} = \frac{1}{\sqrt{7}-2} \cdot \frac{\sqrt{7}+2}{\sqrt{7}+2} = \frac{\sqrt{7}+2}{(\sqrt{7}-2)(\sqrt{7}+2)} = \frac{\sqrt{7}+2}{(\sqrt{7})^2-2^2} = \frac{\sqrt{7}+2}{3}.$$

$$b) \frac{3-\sqrt{2}}{3+\sqrt{2}} = \frac{3-\sqrt{2}}{3+\sqrt{2}} \cdot \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{(3-\sqrt{2})^2}{(3+\sqrt{2})(3-\sqrt{2})} = \frac{9-6\sqrt{2}+2}{3^2-(\sqrt{2})^2} = \frac{11-6\sqrt{2}}{7}.$$

$$c) \frac{1}{2\sqrt{3}+1} = \frac{1}{2\sqrt{3}+1} \cdot \frac{2\sqrt{3}-1}{2\sqrt{3}-1} = \frac{2\sqrt{3}-1}{(2\sqrt{3})^2-1} = \frac{2\sqrt{3}-1}{12-1} = \frac{2\sqrt{3}-1}{11}.$$

**Shembulli 7.**  $\left( \frac{12}{\sqrt{5}+1} + \frac{7}{\sqrt{5}-2} + \frac{4}{3-\sqrt{5}} \right) \cdot \frac{1}{11\sqrt{5}+14}$ .

**Zgjidhja.**

$$\begin{aligned} & \left( \frac{12}{\sqrt{5}+1} + \frac{7}{\sqrt{5}-2} + \frac{4}{3-\sqrt{5}} \right) \cdot \frac{1}{11\sqrt{5}+14} \\ &= \left( \frac{12}{\sqrt{5}+1} \cdot \frac{\sqrt{5}-1}{\sqrt{5}-1} + \frac{7}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2} + \frac{4}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} \right) \cdot \frac{1}{11\sqrt{5}+14} \end{aligned}$$



$$\begin{aligned}
&= \left( \frac{12\sqrt{5}-12}{5-1} + \frac{7\sqrt{5}+14}{5-4} + \frac{12+4\sqrt{5}}{9-5} \right) \cdot \frac{1}{11\sqrt{5}+14} \\
&= \left( \frac{12(\sqrt{5}-1)}{4} + 7\sqrt{5}+14 + (3+\sqrt{5}) \right) \cdot \frac{1}{11\sqrt{5}+14} \\
&= (3\sqrt{5}-3+7\sqrt{5}+14+3+\sqrt{5}) \cdot \frac{1}{11\sqrt{5}+14} \\
&= (11\sqrt{5}+14) \cdot \frac{1}{11\sqrt{5}+14} = 1.
\end{aligned}$$

**Shembulli 8.** a)  $\frac{x+y}{x^{\frac{1}{2}}-y^{\frac{1}{2}}} - \frac{x-y}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}$ ;      b)  $\frac{2x+1}{4^{\frac{1}{3}}x^{\frac{2}{3}}+(2x)^{\frac{1}{3}}+1} + \frac{2x-1}{1-\sqrt[3]{2x}}$ .

**Zgjidhja.**

a) 
$$\begin{aligned}
&\frac{x+y}{x^{\frac{1}{2}}-y^{\frac{1}{2}}} - \frac{x-y}{x^{\frac{1}{2}}+y^{\frac{1}{2}}} = \frac{x+y}{\sqrt{x}-\sqrt{y}} - \frac{x-y}{\sqrt{x}+\sqrt{y}} \\
&= \frac{(x+y)(\sqrt{x}+\sqrt{y}) - (x-y)(\sqrt{x}-\sqrt{y})}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} \\
&= \frac{x\sqrt{x}+x\sqrt{y}+y\sqrt{x}+y\sqrt{y} - (x\sqrt{x}-x\sqrt{y}-y\sqrt{x}+y\sqrt{y})}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} \\
&= \frac{x\sqrt{x}+x\sqrt{y}+y\sqrt{x}+y\sqrt{y} - x\sqrt{x}+x\sqrt{y}+y\sqrt{x}-y\sqrt{y}}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} \\
&= \frac{2x\sqrt{y}+2y\sqrt{x}}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} = \frac{2\sqrt{x}\sqrt{y}(\sqrt{x}+\sqrt{y})}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} = \frac{2\sqrt{xy}}{\sqrt{x}-\sqrt{y}}.
\end{aligned}$$

b) 
$$\begin{aligned}
&\frac{2x+1}{4^{\frac{1}{3}}x^{\frac{2}{3}}+(2x)^{\frac{1}{3}}+1} + \frac{2x-1}{1-\sqrt[3]{2x}} = \frac{2x+1}{(2^2)^{\frac{1}{3}}x^{\frac{2}{3}}+(2x)^{\frac{1}{3}}+1} + \frac{1-2x}{\sqrt[3]{2x}-1} \\
&= \frac{2x+1}{(2x)^{\frac{2}{3}}+(2x)^{\frac{1}{3}}+1} + \frac{1-2x}{(2x)^{\frac{1}{3}}-1}
\end{aligned}$$

$$\begin{aligned} & \frac{(2x+1) \cdot \left( (2x)^{\frac{1}{3}} - 1 \right) + (1-2x) \cdot \left( (2x)^{\frac{2}{3}} + (2x)^{\frac{1}{3}} + 1 \right)}{\left( (2x)^{\frac{1}{3}} - 1 \right) \cdot \left( (2x)^{\frac{2}{3}} + (2x)^{\frac{1}{3}} + 1 \right)} \\ &= \frac{(2x)(2x)^{\frac{1}{3}} - 2x + (2x)^{\frac{1}{3}} - 1 + (2x)^{\frac{2}{3}} + (2x)^{\frac{1}{3}} + 1 - (2x)(2x)^{\frac{2}{3}} - (2x)(2x)^{\frac{1}{3}} - 2x}{\left( (2x)^{\frac{1}{3}} \right)^3 - 1} \\ &= \frac{(2x)^{\frac{2}{3}}(1-2x) + 2(2x)^{\frac{1}{3}} - 4x}{2x-1}. \end{aligned}$$

**Detyra për ushtrime**

Njehsoni:

1. a)  $2^0$ ;      b)  $3^0$ ;      c)  $1^0$ ;      d)  $\left(\frac{1}{4}\right)^0$ ;      e)  $\left(\frac{\sqrt{2}}{3}\right)^0$ ;  
 f)  $\left(\frac{\pi}{3}\right)^0$ ;      g)  $\left(\frac{2\sqrt{3}}{3}\right)^0$ ;      h)  $x^0$ ;      i)  $\left(\frac{x+1}{2}\right)^0$ ;      j)  $\left(\frac{x+\sqrt{2}}{\pi}\right)^0$ .
2. a)  $(x^0 + 2y)^0$ ;      b)  $(x + 2z^0 + 3y^0)^0$ ;      c)  $x^0 + y^0 + (z^2)^0$ .
3. a)  $2(a + b^2)^0$ ;      b)  $(2(a^2 + b)^0)^0$ ;      c)  $\left(\frac{x}{3} + \frac{y}{4^0}\right)^0$ .
4. a)  $x^0 \cdot y^0 + 1$ ;      b)  $x^0 \cdot \frac{3}{4}$ ;      c)  $x^0 : x^2$ ;  
 d)  $x^0 \cdot x^2$ ;      e)  $x^2 : x^0$ ;      f)  $x^2 \cdot x^0 + \frac{1}{2}$ .
5. a)  $(x - 3y)^0$ ;      b)  $x^0 - 3y^0$ ;      c)  $x^0 - (3y)^0$ ;      d)  $x^0 - 3^0 \cdot y$ .
6. a)  $2^{-1}$ ;      b)  $2^{-2}$ ;      c)  $3^{-2}$ ;      d)  $4^{-3}$ ;      e)  $(-2)^{-2}$ .
7. a)  $\frac{1}{2^{-3}}$ ;      b)  $\frac{3}{2^{-4}}$ ;      c)  $\frac{2}{3^{-2}}$ ;      d)  $\frac{-5}{(-4)^{-1}}$ ;      e)  $\frac{a}{a^2}$ .
8. a)  $\left(\frac{1}{2}\right)^{-1}$ ;      b)  $\left(1\frac{1}{2}\right)^{-2}$ ;      c)  $\left(1\frac{3}{4}\right)^{-3}$ ;      d)  $(0.4)^{-1}$ ;      e)  $(-0.5)^{-1}$ .
9. a)  $3 \cdot 2^{-1}$ ;      b)  $2^2 \cdot 3^{-1}$ ;      c)  $2^4 \cdot 4^{-2}$ ;      d)  $8^2 \cdot 4^{-4}$ ;      e)  $\frac{1}{3}\left(\frac{2}{3}\right)^{-2}$ .
10. a)  $x^{-2}$ ;      b)  $x^{-4}$ ;      c)  $\frac{x}{y^{-3}}$ ;      d)  $\frac{3(x+y)}{(x+y)^{-2}}$ .
11. a)  $2^{-2} + 3^{-2} + 1^{-3} + 4^{-2}$ ;      b)  $\left(\frac{1}{2}\right)^{-1} - \left(\frac{1}{3}\right)^{-2} + \left(1\frac{1}{3}\right)^{-3}$ ;  
 c)  $\left(-\frac{1}{3}\right)^{-1} - \left(-\frac{1}{3}\right)^{-2} - \left(-\frac{1}{3}\right)^{-3}$ ;      d)  $\left(-\frac{1}{2}\right)^{-4} + \left(-\frac{1}{2}\right)^{-3} - \left(-\frac{1}{2}\right)^{-2}$ .

12. a)  $((-2)^{-2} \cdot (-3)^{-3}) : (-4)^{-1}$ ;      b)  $(2^5 \cdot (-5)^{-2}) \cdot 25 + \left(2 + \left(\frac{1}{2}\right)^{-3}\right)^0$ ;  
 c)  $\left(1\frac{1}{2} - \left(1\frac{2}{3}\right)^0\right) \cdot \left(1\frac{1}{3} - \left(1\frac{1}{2}\right)^{-2}\right)$ ;  
 d)  $\left(\left(-\frac{1}{3}\right)^{-2} + \left(1\frac{2}{3}\right)^{-3}\right)^{-2} + \left(\left(1\frac{1}{3}\right)^2 - \left(\frac{3}{4}\right)^{-2}\right)$ .

Në shprehjet vijuese të kryhen veprimet ashtu që të liroheni nga fuqitë negative.

13. a)  $ax^{-1}y^{-2}$ ;      b)  $(a^2)^{-3}x^{-2}c^{-3}$ ;      c)  $(ab^2c^3)^{-2}x^{-4}$ .  
 14. a)  $\frac{2}{x^{-2} \cdot 3^{-3} \cdot a^{-4}}$ ;      b)  $\frac{1}{x^{-3}y^{-4}z}$ ;      c)  $\frac{3}{4^{-1}x^{-1}(x+y)^{-2}}$ .  
 15. a)  $\frac{x^{-3}y^{-2}z}{a^{-2}b^{-3}}$ ;      b)  $\frac{x^{-1}y^{-3}(z+2)^{-2}}{3^{-1}x(y+2)^{-1}}$ ;      c)  $\frac{a^{-2}b}{(b+1)^{-3}(a-1)^{-2}}$ .  
 16. a)  $(x-y)^{-3} \cdot \frac{1}{2^{-1} \cdot x^{-3}}$ ;      b)  $\frac{(x-z)^{-2}}{(x+y-z)^{-2}}$ ;      c)  $\frac{x^{-3}a^{-4}b^{-5}}{3^{-1}y^{-2}}$ .

Shprehjet e dhëna të shkruhen në formë të prodhimit të fuqive:

17. a)  $\frac{a}{b^2}$ ;      b)  $\left(\frac{x}{y}\right)^2$ ;      c)  $\frac{1}{x}$ ;      d)  $\frac{1}{a^3}$ .  
 18. a)  $\frac{xy}{z^2}$ ;      b)  $\frac{xy^2}{z^3t^4}$ ;      c)  $\left(\frac{ab}{xyz}\right)^3$ ;      d)  $\frac{ax}{(by)^2}$ .  
 19. a)  $\frac{x}{(x-y)^2}$ ;      b)  $\frac{(x-y)^{-2}}{(x+y)^{-3}}$ ;      c)  $\frac{x}{y^{-2}(x-y)^{-3}}$ ;      d)  $\frac{x^2}{x(x+a)^3}$ .  
 20. a)  $\frac{3x^2y^{-3}}{4z^{-2}(y+1)}$ ;      b)  $\frac{(x-1)^{-3}}{4^{-2}(x-2)^{-5}}$ ;      c)  $\frac{2x^{-3}y}{3x^{-3}t^4}$ ;      d)  $\frac{2(x+y)^2}{4a(x-y)^{-3}}$ .  
 21. a)  $\frac{x}{y^{2n}}$ ;      b)  $\left(\frac{1}{x-y}\right)^{n+1}$ ;      c)  $\frac{x}{y^t z^c}$ ;      d)  $\frac{a^m}{b^n c^{-t}}$ .

Njehsoni:

$$22. \quad a) \quad 2^{-2} + 3^{-1} + \frac{1}{4^{-3}};$$

$$b) \quad \left(\frac{2}{5}\right)^{-1} \cdot \frac{12}{2^{-3}};$$

$$c) \quad \frac{1}{3^{-2}} : \left(\frac{3}{5}\right)^{-3};$$

$$d) \quad 2^{-3} - \left(1\frac{1}{4}\right)^{-2} \cdot (2^{-1})^{-2}.$$

Të kryhen veprimet me fuqi:

$$23. \quad \left(\frac{x^3 \cdot x^2}{x^4}\right)^2 \cdot \frac{x^3 : x^2}{x}.$$

$$24. \quad (0.2x^{-3}y^{-2})^2 \cdot \left(\frac{x^{-2}}{2y^3}\right)^{-2}.$$

$$25. \quad \left(-\frac{7x^2}{3y^4}\right)^{-3} \cdot \left(\frac{9y^2}{49x^4}\right)^{-2}.$$

$$26. \quad \left(\frac{x^3y^{-2}}{9y^2}\right)^{-2} \cdot \left(\frac{x^2y^{-3}}{6y^3}\right)^2.$$

$$27. \quad \left(\frac{8x^2y}{12y^2x} : \frac{xy}{x^3y^2}\right) : \frac{15x^2y^{-2}}{3xy^{-1}}.$$

$$28. \quad \left(\frac{x^5}{y^2}\right)^{-2} \cdot \left(\frac{-2y^2}{x^4}\right)^{-3}.$$

$$29. \quad \left(\frac{-x^3}{y^5}\right)^{-5} \cdot \left(\frac{3y^7}{x^3}\right)^{-2}.$$

$$30. \quad \left(\frac{1}{6}x^{-1}y^3\right)^{-2} \cdot \left(\frac{x^2}{y^2}\right)^{-2} \cdot \left(\frac{2x^2}{y^3}\right)^{-4}.$$

$$31. \quad \left(\frac{1}{2}x^{-1}y^3\right)^{-3} : (x^{-2} : y^{-8}).$$

$$32. \quad \frac{(x^2y^3)^{-8} \cdot (x^4y^6)^6}{(x^4y^6)^2 \cdot (x^{-1}y^{-2})^3} \cdot \frac{((x^2y)^{-4} : (x^5y^3)^2)}{(x^3)^{-5} \cdot y^7} : \left(\frac{1}{y}\right)^{11}.$$

Njehsoni

$$33. \quad a) \quad \sqrt[3]{8 \cdot 27};$$

$$b) \quad \sqrt[3]{2} \cdot \sqrt[3]{4};$$

$$c) \quad \sqrt[5]{6^5 \cdot 3^5};$$

$$d) \quad \sqrt{20} \cdot \sqrt{5};$$

$$e) \quad \sqrt[4]{72 \cdot 18};$$

$$f) \quad \sqrt{27} \cdot \sqrt{12}.$$

$$34. \quad \sqrt[5]{7^3 \cdot 2^2} \cdot \sqrt[5]{7^2 \cdot 2^3}.$$

$$35. \quad \sqrt{8\sqrt{4\sqrt[3]{64}}}.$$

$$36. \quad \sqrt{5(\sqrt{2}-1)^2} + 3\sqrt{(1-\sqrt{2})^2}.$$

$$37. \quad \sqrt{12} + \sqrt{45} + \sqrt{18}.$$

$$38. \quad 5\sqrt{\frac{1}{5}} + \frac{1}{2}\sqrt{20} - \frac{5}{4}\sqrt{20}.$$

$$39. \quad (3\sqrt{5}-2)(3\sqrt{5}-1).$$

$$40. \quad \left(2\sqrt{6} - 4\sqrt{3} + 5\sqrt{2} - \frac{1}{4}\sqrt{8}\right) \cdot 3\sqrt{6}.$$

$$41. (2\sqrt{6} - 3\sqrt{5} + 1) : 3 - (\sqrt{6} + 2\sqrt{5} - 3) : 5 - (3\sqrt{6} + 4\sqrt{5} - 1) : 15.$$

$$42. \left( \frac{1}{2}\sqrt{8} - 3\sqrt{2} + \sqrt{10} \right) \cdot (\sqrt{2} + 3\sqrt{1.6} + 5\sqrt{0.4}).$$

$$43. (12\sqrt{50} - 8\sqrt{200} + 7\sqrt{450}) : \sqrt{10}.$$

$$44. \left( \frac{1}{3}\sqrt{\frac{1}{2}} - \frac{2}{3}\sqrt{\frac{1}{3}} + \frac{2}{7}\sqrt{\frac{4}{5}} \right) : \left( \frac{2}{7}\sqrt{\frac{1}{8}} \right).$$

$$45. (\sqrt{4+2\sqrt{3}} - \sqrt{4-2\sqrt{3}})(\sqrt{4-2\sqrt{3}} + \sqrt{4+2\sqrt{3}}).$$

Njehsoni

$$46. 2\sqrt{18} + 3\sqrt{8} + 3\sqrt{32} - \sqrt{50}.$$

$$47. (2\sqrt{8} + 3\sqrt{5} - 7\sqrt{2})(\sqrt{72} + \sqrt{20} - 4\sqrt{2}).$$

$$48. (4 + \sqrt{6})(3\sqrt{2} - 5\sqrt{3}).$$

Të racionalizohet emëruesi:

$$49. a) \frac{1}{\sqrt{3}};$$

$$b) \frac{\sqrt{3}}{\sqrt[3]{2}};$$

$$c) \frac{2}{\sqrt[4]{49}};$$

$$50. \frac{\sqrt{2}}{\sqrt[3]{3}}.$$

$$51. \frac{1}{\sqrt{2} - \sqrt{3}}.$$

$$52. \frac{2}{2\sqrt{2} - 1}.$$

$$53. \frac{1}{2\sqrt{2} + 3\sqrt{3}}.$$

$$54. \frac{1}{\sqrt{2} + \sqrt{3}}.$$

$$55. \frac{4}{3\sqrt{2} + 1}.$$

$$56. \frac{1}{2\sqrt{2} - 3\sqrt{3}}.$$